

AMENDMENTS TO THE CLAIMS

This listing of claims will replace all prior versions, and listings, of claims in the application:

Listing of Claims:

- 1 1. (Currently amended) A method for improving computation efficiency in
2 solving using a computer system to solve a global optimization problem specified
3 by a function f and a set of equality constraints, the method comprising:
4 receiving a representation of the function f and the set of equality
5 constraints $q_i(\mathbf{x}) = 0$ ($i=1, \dots, r$) at the computer system, wherein f is a scalar
6 function of a vector $\mathbf{x} = (x_1, x_2, x_3, \dots, x_n)$;
7 storing the representation in a memory within the computer system;
8 performing an interval global optimization process to compute guaranteed
9 bounds on a globally minimum value of the function $f(\mathbf{x})$ subject to the set of
10 equality constraints with improved computation efficiency;
11 wherein performing computations during interval global optimization
12 process involves using a special-purpose interval arithmetic unit for interval
13 computations;
14 wherein performing the interval global optimization process involves,
15 applying term consistency to the set of equality constraints
16 over a subbox \mathbf{X} , and
17 excluding portions of the subbox \mathbf{X} that can be shown to
18 violate any of the equality constraints;
19 wherein applying term consistency involves:
20 symbolically manipulating an equation within the computer
21 system to solve for a term, $g(x_i)$, thereby producing a modified

22 equation $g(x_j) = h(\mathbf{x})$, wherein the term $g(x_j)$ can be analytically
 23 inverted to produce an inverse function $g^{-1}(y)$;
 24 substituting the subbox \mathbf{X} into the modified equation to
 25 produce the equation $g(X'_j) = h(\mathbf{X})$;
 26 solving for $X'_j = g^{-1}(h(\mathbf{X}))$; and
 27 intersecting X'_j with the interval X_j to produce a new
 28 subbox \mathbf{X}^+ ;
 29 wherein the new subbox \mathbf{X}^+ contains all solutions of the
 30 equation within the subbox \mathbf{X} , and wherein the size of the new
 31 subbox \mathbf{X}^+ is less than or equal to the size of the subbox \mathbf{X} .

1 2. (Original) The method of claim 1, wherein performing the interval
 2 global optimization process involves:
 3 preconditioning the set of equality constraints through multiplication by an
 4 approximate inverse matrix \mathbf{B} to produce a set of preconditioned equality
 5 constraints;
 6 applying term consistency to the set of preconditioned equality constraints
 7 over the subbox \mathbf{X} ; and
 8 excluding portions of the subbox \mathbf{X} that can be shown to violate any of the
 9 preconditioned equality constraints.

1 3. (Original) The method of claim 1, wherein performing the interval
 2 global optimization process involves:
 3 keeping track of a least upper bound f_bar of the function $f(\mathbf{x})$;
 4 unconditionally removing from consideration any subbox for which
 5 $\inf(f(\mathbf{x})) > f_bar$;
 6 applying term consistency to the inequality $f(\mathbf{x}) \# f_bar$ over the subbox \mathbf{X} ;
 7 and

8 excluding portions of the subbox **X** that violate the inequality.

1 4. (Canceled)

1 5. (Original) The method of claim 1, wherein performing the interval
2 global optimization process involves:
3 applying box consistency to the set of equality constraints $q_i(\mathbf{x}) = 0$
4 ($i=1, \dots, r$) over the subbox **X**; and
5 excluding portions of the subbox **X** that violate the set of equality
6 constraints.

1 6. (Original) The method of claim 1, wherein performing the interval
2 global optimization process involves:
3 evaluating a first termination condition;
4 wherein the first termination condition is TRUE if a function of the width
5 of the subbox **X** is less than a pre-specified value, ε_X , and the absolute value of the
6 function, f , over the subbox **X** is less than a pre-specified value, ε_F ; and
7 if the first termination condition is TRUE, terminating further splitting of
8 the subbox **X**.

1 7. (Original) The method of claim 1, wherein performing the interval
2 global optimization process involves performing an interval Newton step on the
3 John conditions.

1 8. (Currently amended) A computer-readable storage medium storing
2 instructions that when executed by a computer system cause the computer system
3 | to perform a method for improving computation efficiency in solving using a

4 | ~~computer system to solve a global optimization problem specified by a function f~~
 5 | and a set of equality constraints, the method comprising:
 6 | receiving a representation of the function f and the set of equality
 7 | constraints $q_i(\mathbf{x}) = 0$ ($i=1, \dots, r$) at the computer system, wherein f is a scalar
 8 | function of a vector $\mathbf{x} = (x_1, x_2, x_3, \dots, x_n)$;
 9 | storing the representation in a memory within the computer system;
 10 | performing an interval global optimization process to compute guaranteed
 11 | bounds on a globally minimum value of the function $f(\mathbf{x})$ subject to the set of
 12 | equality constraints with improved computation efficiency;
 13 | wherein performing computations during interval global optimization
 14 | process involves using a special-purpose interval arithmetic unit for interval
 15 | computations;
 16 | wherein performing the interval global optimization process involves,
 17 | applying term consistency to the set of equality constraints
 18 | over a subbox \mathbf{X} , and
 19 | excluding portions of the subbox \mathbf{X} that can be shown to
 20 | violate any of the equality constraints;
 21 | wherein applying term consistency involves:
 22 | symbolically manipulating an equation within the computer
 23 | system to solve for a term, $g(x_j)$, thereby producing a modified
 24 | equation $g(x_j) = h(\mathbf{x})$, wherein the term $g(x_j)$ can be analytically
 25 | inverted to produce an inverse function $g^{-1}(y)$;
 26 | substituting the subbox \mathbf{X} into the modified equation to
 27 | produce the equation $g(X'_j) = h(\mathbf{X})$;
 28 | solving for $X'_j = g^{-1}(h(\mathbf{X}))$; and
 29 | intersecting X'_j with the interval X_j to produce a new
 30 | subbox \mathbf{X}^+ ;

31 wherein the new subbox X^+ contains all solutions of the
32 equation within the subbox X , and wherein the size of the new
33 subbox X^+ is less than or equal to the size of the subbox X .

1 9. (Original) The computer-readable storage medium of claim 8, wherein
2 performing the interval global optimization process involves:
3 preconditioning the set of equality constraints through multiplication by an
4 approximate inverse matrix B to produce a set of preconditioned equality
5 constraints;
6 applying term consistency to the set of preconditioned equality constraints
7 over the subbox X ; and
8 excluding portions of the subbox X that can be shown to violate any of the
9 preconditioned equality constraints.

1 10. (Original) The computer-readable storage medium of claim 8, wherein
2 performing the interval global optimization process involves:
3 keeping track of a least upper bound f_bar of the function $f(x)$;
4 unconditionally removing from consideration any subbox for which
5 $\inf(f(x)) > f_bar$;
6 applying term consistency to the inequality $f(x) \# f_bar$ over the subbox X ;
7 and
8 excluding portions of the subbox X that violate the inequality.

1 11. (Canceled)

1 12. (Original) The computer-readable storage medium of claim 8, wherein
2 performing the interval global optimization process involves:

3 applying box consistency to the set of equality constraints $q_i(\mathbf{x}) = 0$
4 ($i=1, \dots, r$) over the subbox \mathbf{X} ; and
5 excluding portions of the subbox \mathbf{X} that violate the set of equality
6 constraints.

1 13. (Original) The computer-readable storage medium of claim 8, wherein
2 performing the interval global optimization process involves:
3 evaluating a first termination condition;
4 wherein the first termination condition is TRUE if a function of the width
5 of the subbox \mathbf{X} is less than a pre-specified value, ε_X , and the absolute value of the
6 function, f , over the subbox \mathbf{X} is less than a pre-specified value, ε_F ; and
7 if the first termination condition is TRUE, terminating further splitting of
8 the subbox \mathbf{X} .

1 14. (Original) The computer-readable storage medium of claim 8, wherein
2 performing the interval global optimization process involves performing an
3 interval Newton step on the John conditions.

1 15. (Currently amended) An apparatus that improves computation
2 efficiency in solving solves a global optimization problem specified by a function
3 f and a set of equality constraints, the apparatus comprising:
4 a receiving mechanism that is configured to receive a representation of the
5 function f and the set of equality constraints $q_i(\mathbf{x}) = 0$ ($i=1, \dots, r$), wherein f is a
6 scalar function of a vector $\mathbf{x} = (x_1, x_2, x_3, \dots, x_n)$;
7 a memory for storing the representation;
8 an optimizer that is configured to perform an interval global optimization
9 process to compute guaranteed bounds on a globally minimum value of the

10 function $f(\mathbf{x})$ subject to the set of equality constraints with improved computation
 11 efficiency;
 12 wherein the optimizer configured to perform computations during interval
 13 global optimization process involves using a special-purpose interval arithmetic
 14 unit for interval computations;
 15 wherein the optimizer is configured to,
 16 apply term consistency to the set of equality constraints
 17 over a subbox \mathbf{X} , and to
 18 exclude portions of the subbox \mathbf{X} that can be shown to
 19 violate any of the equality constraints;
 20 wherein while applying term consistency, the optimizer is configured to:
 21 symbolically manipulate an equation to solve for a term,
 22 $g(x_j)$, thereby producing a modified equation $g(x_j) = h(\mathbf{x})$, wherein
 23 the term $g(x_j)$ can be analytically inverted to produce an inverse
 24 function $g^{-1}(y)$;
 25 substitute the subbox \mathbf{X} into the modified equation to
 26 produce the equation $g(X'_j) = h(\mathbf{X})$;
 27 solve for $X'_j = g^{-1}(h(\mathbf{X}))$; and to
 28 intersect X'_j with the interval X_j to produce a new
 29 subbox \mathbf{X}^+ ;
 30 wherein the new subbox \mathbf{X}^+ contains all solutions of the
 31 equation within the subbox \mathbf{X} , and wherein the size of the new
 32 subbox \mathbf{X}^+ is less than or equal to the size of the subbox \mathbf{X} .

1 16. (Original) The apparatus of claim 15, wherein the optimizer is
 2 configured to:

3 precondition the set of equality constraints through multiplication by an
4 approximate inverse matrix **B** to produce a set of preconditioned equality
5 constraints;
6 apply term consistency to the set of preconditioned equality constraints
7 over the subbox **X**; and to
8 exclude portions of the subbox **X** that can be shown to violate any of the
9 preconditioned equality constraints.

1 17. (Original) The apparatus of claim 15, wherein the optimizer is
2 configured to:
3 keep track of a least upper bound f_bar of the function $f(\mathbf{x})$;
4 unconditionally remove from consideration any subbox for which
5 $\inf(f(\mathbf{x})) > f_bar$;
6 apply term consistency to the inequality $f(\mathbf{x}) \# f_bar$ over the subbox **X**;
7 and to
8 exclude portions of the subbox **X** that violate the inequality.

1 18. (Canceled)

1 19. (Original) The apparatus of claim 15, wherein the optimizer is
2 configured to:
3 apply box consistency to the set of equality constraints $q_i(\mathbf{x}) = 0$ ($i=1, \dots, r$)
4 over the subbox **X**; and to
5 exclude portions of the subbox **X** that violate the set of equality
6 constraints.

1 20. (Original) The apparatus of claim 15, wherein the optimizer is
2 configured to:

3 evaluate a first termination condition;
4 wherein the first termination condition is TRUE if a function of the width
5 of the subbox **X** is less than a pre-specified value, ε_X , and the absolute value of the
6 function, f , over the subbox **X** is less than a pre-specified value, ε_F ; and to
7 terminate further splitting of the subbox **X** if the first termination
8 condition is TRUE

1 21. (Original) The apparatus of claim 15, wherein the optimizer is
2 configured to perform an interval Newton step on the John conditions.